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SUPERSYMMETRIC STRING WAVES

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Abstract

We present plane-wave-type solutions to the superstring effective action which have unbroken space-time supersymmetries. They describe dilaton, axion and gauge fields in a generalization of the Brinkmann metric. A crucial property of the solutions is a conspiracy between the metric and the axion field. Furthermore, due to a relation between the geometry and the gauge fields, the α' string corrections to the effective on-shell action and to the solutions themselves vanish. We call these solutions supersymmetric string waves.

1. Introduction

Soon after the discovery of general relativity by Einstein, it became clear that the field equations of gravity admit solutions that describe fascinating new phenomena. Examples of such solutions are black holes, the expanding universe and gravitational waves (for a review of exact solutions of Einstein's field equations, see [1]). These and other solutions have since then become an essential aspect of astrophysics and cosmology. Besides gravity, the only other long range force known to exist in nature is the electromagnetic force, whose proper formulation was given by Maxwell. The Einstein-Maxwell theory, which describes the interplay between gravity and electromagnetism, admits solutions such as charged black holes.

Recently, Einstein-Maxwell theories have been considered that include additional scalar or antisymmetric tensor fields or in which the electromagnetic field is replaced by non-Abelian Yang-Mills fields. These modified Einstein-Maxwell theories admit new solutions whose consistency crucially depends on the presence of the new fields and/or on the non-Abelian nature of the Yang-Mills fields. Examples of such new solutions are wormholes (solutions that describe a topology change of spacetime), black strings (string-like solutions which, similar to black holes, are hidden behind an event horizon), and five-brane solitons (magnetic monopole-type solutions with five spacelike internal dimensions). For recent reviews, see.[2 3]

2. Superstring Effective Actions

One motivation for studying the above-mentioned modifications to Einstein-Maxwell theory is that they arise in string theory. In string theory elementary particles are described by the excitations of a string, rather than by points. These excitations also include the gravitational field, so that string theory describes gravitational and other forces on an equal footing. The size of the string can be characterized by a parameter α', in such a way that in the limit $\alpha' \to 0$, the so-called zero-slope limit, a theory of point particles is obtained. Successive approximations to string theory can be found by considering the expansion in powers of α'. This expansion gives rise to so-called "effective actions" for string theory. The zero-slope limit $\alpha' \to 0$ corresponds to a modified Einstein-Maxwell theory of the type discussed above. The complete effective action also includes contributions which are of higher order in the Riemann tensor (which describes the curvature of space-time) and the Yang-Mills field strength.

To be more specific, we consider the ten-dimensional heterotic superstring. The bosonic part of the zero-slope limit is then given by

$$\mathcal{L}(\text{zero}-\text{slope}) = \frac{1}{2}\sqrt{-g}e^{-2\phi}\Big(-R+4(\partial\phi)^2-\frac{1}{3}H^2\Big)\,. \tag{1}$$

Here $g_{\mu\nu}$ is the gravitational field, ϕ is the dilaton and $B_{\mu\nu}$, with field-strength $H = dB$, is the so-called axion. An additional Yang-Mills gauge field V_μ is introduced at order α'. Schematically, this leads to an effective action of the form

$$\mathcal{L}(\text{effective}) = \mathcal{L}(\text{zero}-\text{slope}) - \frac{1}{30}\alpha'(F^2-R^2)+O(\alpha'^2)\,, \tag{2}$$

where it is understood that everywhere the field-strenth H receives implicit α' corrections involving Chern-Simons forms:

$$H = dB + \alpha'\Big[\text{Yang}-\text{Mills} + \text{Lorentz Chern}-\text{Simons form}\Big]\,. \tag{3}$$

More information about the explicit form of the higher-order terms can be obtained by requiring supersymmetry of the effective action (see e.g.[4] and the talk by H. Suelmann).

3. Supersymmetric String Waves

At this time, only a few solutions to the string equations of motion corresponding to the Lagrangian (2) are known. In this talk I will describe a new class of solutions, called supersymmetric string waves (SSW), which was found recently in.[5]

The gravitational field corresponding to the SSW solution is given by

$$g_{\mu\nu} = \eta_{\mu\nu} + A_{(\mu}(u,x^i)l_{\nu)}\,, \tag{4}$$

where we have introduced lightcone coordinates $x^\mu = \{u, v, x^i\}$ $(i = 1, \cdots, 8)$. Here l_μ is a covariantly constant null vector:

$$\nabla_\mu l_\nu = 0\,, \qquad\qquad l^\mu l_\mu = 0\,. \tag{5}$$

The vector function $A_\mu(u, x^i)$ satisfies the constraint $l^\mu A_\mu(u, x^i) = 0$ and has zero laplacian, otherwise it is arbitrary. For instance, vector functions of the form $A_\mu(u, x^i) = l_\mu f_{ij} x^i x^j$ describe exact plane waves while functions of the form $A_\mu(u, x^i) = l_\mu \delta(u) f(x^i)$ describe shock waves. Exact plane waves as solutions to the string equations of motion have been considered in.[6] The general solution (4) was considered in.[7] We have generalized these results as follows.[5]

First of all, the following non-trivial dilaton, axion and Yang-Mills background can be included

$$\begin{aligned} \phi &= \phi(u)\,, \\ H_{\mu\nu\rho} &= l_{[\mu}\mathcal{A}_{\nu\rho]}\,, \qquad\qquad \mathcal{A}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \\ F^{ij}_{\mu\nu} &= l_{[\mu}\partial_{\nu]}\mathcal{A}^{ij}\,, \end{aligned} \tag{6}$$

such that *all* α' string corrections to the on-shell effective action action and to the solutions themselves vanish. To show this, we use the fact that the α' string corrections occur in the form of so-called T-tensors such as:[4]

$$T_{\mu\nu\rho\sigma} = \alpha'\left(R_{[\mu\nu}{}^{ab}R_{\lambda\rho]}{}^{ab} + \frac{1}{30}\mathrm{tr}F_{[\mu\nu}F_{\lambda\rho]}\right). \tag{7}$$

Due to the particular relation between the metric and the gauge fields occurring in the SSW solution all T-tensors and therefore all α' corrections vanish.

Secondly, the SSW solutions have eight unbroken supersymmetries which are characterized by the condition $l^\mu\gamma_\mu\epsilon = 0$. These unbroken supersymmetries are due to a conspiracy between the metric and the axion field. More specifically, it turns out that the spin-connection and the field-strength of the axion always occur in the following combinations:

$$\Omega^{ab}_{\pm\mu} = \omega^{ab}_\mu(e) \pm H_\mu{}^{ab}\,. \tag{8}$$

For the SSW solutions the torsion-full connections Ω satisfy the identities $l_a\Omega^{ab}_{+\mu} = l^\mu\Omega^{ab}_{-\mu} = 0$ which play a crucial rule in showing the supersymmetry of the SSW solutions. For more details we refer to.[5]

4. Discussion

Obtaining the general solution to the string equations of motion is a formidable problem. Fortunately, there exist transformations which generate new solutions from old ones. Examples of solution generating transformations are the noncompact symmetry transformations [8] and target space duality transformations.[9] For instance, it has been established that extremal black strings (which are charged black strings with equal charge and mass per unit length) are dual to plane-fronted waves describing strings moving at the speed of light.[10] The SSW solutions are a generalization of the plane-fronted waves. It would therefore be interesting to apply a target space duality transformation on the SSW solutions and see whether one may generate interesting new extremal black string solutions.

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